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THE ECONOMICS OF PARALLELING PIPE

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HYDRAULICS DIVISION

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THE ECONOMICS OF PARALLELING PIPE

George A. Whetstone,[1] A.M. ASCE

SYNOPSIS

In the design of high-head hydroelectric developments and in other engineering applications involving large pipe carrying high pressures the question of relative economy of one large pipe or several smaller pipes often arises. This paper compares the relative total weights of material and the relative diameters for equal pipe stresses in systems of 1, 2, 3, 4, or 5 pipes in parallel.

Consideration is given to the differences resulting from the regime of flow: laminar, partially turbulent (hydraulically smooth pipe), or fully turbulent (hydraulically rough pipe). A brief summary of some of the factors other than weight which affect the final choice is also given.

INTRODUCTION

In problems of transmission of fluids under high pressure such as sometimes occur in water supply or petroleum applications, and which often occur in high-head hydro power developments, the designer may be faced with a choice between the selection of a single pipe or of several pipes in parallel.

Many of the parallel-pipe systems in existence "just grew". The project may have been designed with a single pipe adequate to accomplish its expected purpose. However, as the load grew it became necessary to add one or more pipes in parallel to provide for the required increase in flow. In other cases the ultimate growth was foreseen, but its predicted attainment was well in the future, and it seemed wiser to defer the capital outlay for part of the project. Other developments, particularly older power projects, employed a multiplicity of penstocks due to the limitations in the strength of a single pipe at the time of their construction. With modern pipe few such limitations remain.

Notation

The following notation will be employed in this paper.

A: Cross-sectional area of pipe (square feet)
D: Internal diameter of pipe (feet)
f : Coefficient in the Darcy equation for head lost in pipe friction:

$$h_f = f \frac{L}{D} \frac{v^2}{2g}$$

g: Acceleration of gravity (= 32.2 feet per second per second)
L: Pipe length (feet)
n: Number of parallel pipes in question
p: Working pressure in the fluid (pounds per square inch)
Q: Discharge (cubic feet per second)

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R: Reynolds number
S_t: Tensile stress in the pipe walls (pounds per square inch)
t : Thickness of pipe walls (feet)
v: Fluid velocity in the pipe (feet per second)
w: Weight per foot of pipe (pounds per foot)
ϵ: Nikuradse equivalent sand size roughness (feet)

Relative Weights and Diameters of One and of n Pipes

The most important consideration in pipeline economics is weight, for the cost of the pipe is approximately proportional to the weight. For the usual classes of pipe, which are not pre-stressed by means of bands or cable slings placed on the pipe before its expansion to final size in a hydraulic press, the diameter is proportional to the wall thickness, a fact which appears immediately from the standard formula for stress in a pipe wall

$$\frac{t}{D} = \frac{P}{2 S_t} . \tag{1}$$

A comparison will be made of the weight per running foot and of the diameters for two "equivalent" systems, the first consisting of a single pipe (subscript 1), and the other consisting of n identical pipes (subscript n) of the same material as the single pipe.

By "equivalent" it shall be understood that both systems have the same length, the same total discharge, the same gross head, and the same net head. This then implies the same friction head.

From the postulated equality of head losses

$$f_1 \frac{L}{D_1} \frac{V_1^2}{2g} = f_n \frac{L}{D_n} \frac{V_n^2}{2g} . \tag{2}$$

But

$$V_1 = \frac{Q}{\frac{\pi}{4} D_1^2} \quad \text{and} \quad V_n = \frac{\frac{Q}{n}}{\frac{\pi}{4} D_n^2} ,$$

whence

$$\frac{f_1}{D_1^5} = \frac{f_n}{n^2 D_n^5} ,$$

or

$$\frac{D_n}{D_1} = \left(\frac{f_n}{n^2 f_1} \right)^{1/5} . \tag{3}$$

The ratio of total areas would be

$$\frac{n A_n}{A_1} = n \left(\frac{D_n}{D_1} \right)^2 = n \left(\frac{f_n}{n^2 f_1} \right)^{2/5} = \left(\frac{n f_n^2}{f_1^2} \right)^{1/5} . \tag{4}$$

Then, for the same total discharge in each system, the velocity ratios would be

$$\frac{V_n}{V_1} = \frac{A_1}{n A_n} = \left(\frac{f_1^2}{n f_n^2} \right)^{1/5}. \qquad (5)$$

Since both systems of pipe are subject to equal stress to within the small differences in the velocity heads, equation (1) would yield

$$\frac{t_n}{t_1} = \frac{D_n}{D_1} = \left(\frac{f_n}{n^2 f_1} \right)^{1/5}. \qquad (6)$$

Thus the comparative weights would be

$$\frac{W_n}{W_1} = \frac{n\pi \left[\left(\frac{D_n}{2} + t_n \right)^2 - \left(\frac{D_n}{2} \right)^2 \right]}{\pi \left[\left(\frac{D_1}{2} + t_1 \right)^2 - \left(\frac{D_1}{2} \right)^2 \right]} = n \left(\frac{t_n}{t_1} \right)^2 = \left(\frac{n f_n^2}{f_1^2} \right)^{1/5}. \qquad (7)$$

To make these expressions more useful, recourse must be had to some expressions for f. Here one encounters the situation that three different relations are required to express f as a function of Reynolds number in the laminar, partially turbulent, and fully turbulent conditions of flow.

Since the kinematic viscosity of the liquid would be the same whether the liquid were to flow in a large pipe or in several small pipes, it follows that

$$\frac{R_n}{R_1} = \frac{V_n D_n}{V_1 D_1} = \left(\frac{f_1}{n^3 f_n} \right)^{1/5}. \qquad (8)$$

The friction relations may now be written in terms of n as follows.

Laminar Flow in Both Systems

Here, from the standard relation f = 64/R,

$$\frac{f_n}{f_1} = \frac{R_1}{R_n} = n^{3/5} \left(\frac{f_n}{f_1} \right)^{1/5}$$

or

$$\frac{f_n}{f_1} = n^{3/4}. \qquad (9)$$

Thus if both flows were laminar, as could easily occur with petroleum products,

$$\frac{W_n}{W_1} = n^{1/5} \left(n^{3/4} \right)^{2/5} = n^{1/2}, \qquad (10)$$

and

$$\frac{D_n}{D_1} = \left(\frac{1}{n}\right)^{2/5}\left(n^{3/4}\right)^{1/5} = \left(\frac{1}{n}\right)^{1/4}. \tag{11}$$

Partially Turbulent Flow in Both Systems

In the range of partial turbulence, which is unfortunately very common in practice, a wide choice of formulas is offered for f. Of these, the Lees formula with the constants determined by Harris[2]

$$f = 0.0061 + \frac{0.55}{R^{1/3}} \tag{12}$$

seems as well authenticated as any. Using this relation one obtains

$$\frac{f_n - 0.0061}{f_1 - 0.0061} = \left(\frac{R_1}{R_n}\right)^{1/3} = \left(\frac{n^3 f_n}{f_1}\right)^{\frac{1}{15}}. \tag{13}$$

Equation (13) evidently has no simple analytical solution for f_n/f_1 as a function of n. However, since n must be an integer, and since f_1 has a rather restricted range of values in practice, the information obtainable from equation (13) is adequately represented by table I and figures 1, 2, and 3.

Fully Turbulent Flow in Both Systems

For this case the Nikuradse rough-pipe law[3]

$$\frac{1}{2\sqrt{f}} = 0.87 - \log_{10}\frac{2\epsilon}{D} \tag{14}$$

seems to be accorded wide acceptance. Since ϵ represents the absolute magnitude of surface roughness particles, it is reasonable to assume that it will remain essentially constant for the various diameters of pipe, all of which are rolled from plates of comparable surface finish.

Thus, rewriting (14) as

$$10^{\left(\frac{1}{2\sqrt{f}} - 0.87\right)} = \frac{D}{2\epsilon},$$

inserting subscripts, and dividing, ϵ being constant,

$$10^{\left(\frac{1}{2\sqrt{f_1}} - \frac{1}{2\sqrt{f_n}}\right)} = \frac{D_1}{D_n},$$

whence

$$\frac{1}{\sqrt{f_1}} - \frac{1}{\sqrt{f_n}} = \frac{2}{5}\log\left(\frac{n^2 f_1}{f_n}\right). \tag{15}$$

2. HARRIS, Charles W.: An Engineering Concept of Flow in Pipes. Transactions, American Society of Civil Engineers. Vol. 115 (1950), page 919.
3. Loc. cit. Discussion by Ralph W. Powell. Page 934.

A tabulated solution for f_n in terms of f_1 together with the solutions obtained by using these results in equations (3) and (7) appears in table II and figures 4, 5, and 6.

Flow of Different Regimes in the Two Systems

Having established relations between the f's and consequently between the diameters and weights, in all cases where flow in both systems is of the same regime, there remains the question of unlike regimes; suppose flow in a single pipe were fully turbulent whereas flow in n pipes were partially turbulent or laminar.

In figure 7 is sketched on a log f - log R grid the friction relations utilized here. The points A, B indicate the change in f and R when going from one to n pipes in laminar flow. C, D and E, F are the corresponding pairs of points for partially and fully developed turbulent flow respectively. It will be remembered from equation (9) and from tables I and II that f_n is always greater than f_1 for the cases considered so far.

It is possible when E is sufficiently close to the Lees-Harris line that point F will lie on this line rather than on a fully-turbulent horizontal. It is also possible, for small values of f, that C can lie on the Lees-Harris line and that D will lie on a horizontal above and to the right of it.

From the tables it may be seen that for $f_1 \geq 0.009$ the ratio f_n/f_1 is greater for partial turbulence than for full turbulence. Thus the ratios w_n/w_1 and D_n/D_1 for partial turbulence provide an upper limit and those for full turbulence a lower limit for a given f_1 and n where the possibility of different regimes exists for values of $f_1 \geq 0.009$.

While it is theoretically possible that flow in a single pipe could be partially or fully turbulent, while that in multiple pipes would be laminar, the uncertainty of the position of the transition deprives a detailed analysis of this situation of any real meaning.

Practical Considerations

By theoretical considerations based on the Lees-Harris smooth-pipe law and the Nikuradse rough-pipe law it has been shown that for "equivalent" pipe systems both working at the same stress ratio, p/S_t, the ratios of diameters and weights for usual values of the friction factor, f_1, in the single pipe are about as given in the following table.

n	2	3	4	5
w_n/w_1	1.16–1.21	1.28–1.36	1.36–1.47	1.43–1.56
D_n/D_1	0.76–0.78	0.65–0.67	0.58–0.61	0.53–0.56

The wall thicknesses would be in the same ratios as the diameters.

These same relations would be valid for the comparison of a penstock consisting of elements of different sizes with a system of n smaller parallel penstocks provided these latter were also to consist of different sized elements with the same size-ratios as the single pipe. Since pressure varies with length in the same manner for each solution, equation (1) would indicate that the changes of pipe size should occur at the same places, and thus that the ratios would apply segment by segment.

It is thus evident that the material used in a single pipe-line would cost less than the material required by n pipelines for the same duty; not so much less, however, but that other features might sometimes make a multiple-pipe

system preferable. Some of these other features which enter into the calculations are mentioned below.

Deferred Construction

Probably the most common reason for constructing parallel pipe systems has been the installation of projects by stages. In a power development it is common to install only part of a project until growth of load justifies the entire development. The interval may be several years during which time the interest on an excessively large pipe might well exceed the saving on materials resulting from constructing a single pipe to carry the ultimate design discharge. In this connection it should also be remembered that a pipeline will wear out about as fast at part load as at full load.

Security

If a plant or a water distribution system depends on a single pipeline for its entire supply any outage of the pipeline due to a break, due to shutdown for inspection or renewal of the corrosion resisting lining, or due to any other cause leads to a shutdown of the plant. The importance of this consideration depends on the availability of standby power interconnected to the same grid or on the storage facilities at the pipe line terminal as the case may be. The additional security of two lines may have a real financial value, especially if they are so equipped with valves that a short reach of either can be cut out with the combined flow passing through the corresponding short reach of the other.

Corrosion Resistance

In the treatment for the prevention of corrosion by metallization with zinc and aluminum, or by painting with any of the preparations designed for this purpose, two factors pull in opposite directions. A single pipe has the advantage of less total surface to be covered, of greater working convenience if the preparation is to be applied inside the pipe, and of less diminution of cross-sectional area due to the thickness of the anti-corrosive layer. Multiple pipes have the advantage that any one of them is subject to retirement from service for short periods without major interference with the functioning of the development.

Pipe Strength in Larger Sizes[4,5]

Modern pressure pipe is welded. That destined for moderate working pressures may then be expanded beyond its elastic limit in a hydraulic press to increase its strength by the resulting cold-working. Modern European practice for high pressures such as occur in high-head hydro plant installations now involve cold-working expansion together with reinforcement by rigid hoops or by cable in individual slings or in continuous spirals.

Thickness of the pipe, or of the liner in the case of reinforced pipe, is limited to the thickness which can be successfully welded. This may in some cases limit the diameter since pipe thickness is proportional to diameter as has been seen previously.

Ease of Construction

Here again two conflicting tendencies exist. Smaller thinner-walled pipe

4. FERRAND, Georges: Pressure Pipe Lines. Water Power. Vol. 2 (1950), pages 52-59.
5. WHETSTONE, George A.: Improved Penstocks Reduce Costs for High-Head Hydro Power. Power Engineering. Vol. 55 (August 1951), pages 64-69.

is easier to handle on the steep mountain sides where power penstocks are usually found. It is easier to join in the field. The stresses on anchorages and supports are in general proportional to the square of the diameter[6]. On the other hand, a multiplicity of pipelines require more excavation, more anchorages, and a wider right-of-way than a single line. The laying of several pipelines may require the provision of a cableway with movable, rather than fixed, end supports.

Commercial Sizes (Diameters and Thicknesses)

For a given discharge and pressure the advantage here may lie with the single pipe or with some other value of n. In any case the point must be considered in the design of an actual project.

SUMMARY

In this paper a fairly definite relation between the friction factor in a single pipe and that in each of a system of "equivalent" pipes has been worked out. Using this, ratios of weights and diameters have been computed. The other factors affecting relative costs of single or of multiple pipe systems have been discussed briefly. No final answer stating that a single line is always to be preferred has been reached, nor can it be, but the writer hopes that the basis for comparison has been made clearer and that preliminary estimates may be more easily obtained by use of the tables presented here.

6. PIPPARD, Alfred J. S.: Stresses in a Restrained Pipe Line. Journal, Institution of Civil Engineers. Vol. 35 (1951), pages 170-184.

TABLE I

Weight and Diameter Ratios for Partial Turbulence

in Both Systems

f_1	f_2	w_2/w_1	D_2/D_1	f_3	w_3/w_1	D_3/D_1	f_4	w_4/w_1	D_4/D_1	f_5	w_5/w_1	D_5/D_1
0.008	0.00829	1.165	0.7632	0.00848	1.275	0.6519	0.00862	1.360	0.5830	0.00874	1.429	0.5347
0.009	0.00944	1.171	0.7651	0.00973	1.285	0.6545	0.00995	1.376	0.5860	0.01014	1.447	0.5380
0.010	0.01060	1.176	0.7667	0.01099	1.294	0.6567	0.01129	1.385	0.5885	0.01154	1.461	0.5406
0.011	0.01175	1.180	0.7679	0.01225	1.301	0.6584	0.01262	1.394	0.5904	0.01294	1.472	0.5427
0.012	0.01291	1.183	0.7690	0.01351	1.306	0.6599	0.01396	1.402	0.5920	0.01434	1.481	0.5444
0.013	0.01406	1.185	0.7698	0.01477	1.311	0.6611	0.01530	1.408	0.5934	0.01574	1.489	0.5460
0.014	0.01522	1.188	0.7706	0.01603	1.315	0.6621	0.01664	1.414	0.5945	0.01715	1.496	0.5471
0.015	0.01638	1.190	0.7713	0.01729	1.319	0.6630	0.01798	1.419	0.5956	0.01855	1.502	0.5481
0.016	0.01754	1.192	0.7719	0.01856	1.322	0.6638	0.01933	1.423	0.5965	0.01996	1.507	0.5491
0.017	0.01870	1.193	0.7725	0.01982	1.325	0.6645	0.02067	1.427	0.5972	0.02137	1.512	0.5499
0.018	0.01986	1.195	0.7729	0.02108	1.327	0.6651	0.02201	1.430	0.5979	0.02278	1.516	0.5506

TABLE I (Continued)

f_1	f_2	w_2/w_1	D_2/D_1	f_3	w_3/w_1	D_3/D_1	f_4	w_4/w_1	D_4/D_1	f_5	w_5/w_1	D_5/D_1
0.019	0.02102	1.196	0.7733	0.02234	1.329	0.6656	0.02335	1.433	0.5985	0.02419	1.520	0.5513
0.020	0.02218	1.197	0.7737	0.02361	1.331	0.6662	0.02470	1.436	0.5991	0.02560	1.523	0.5519
0.022	0.02449	1.199	0.7743	0.02613	1.334	0.6670	0.02739	1.440	0.6001	0.02842	1.529	0.5529
0.024	0.02681	1.201	0.7748	0.02866	1.337	0.6677	0.03008	1.444	0.6009	0.03124	1.533	0.5538
0.026	0.02913	1.202	0.7753	0.03119	1.340	0.6683	0.03277	1.448	0.6016	0.03406	1.537	0.5545
0.028	0.03145	1.203	0.7757	0.03372	1.342	0.6688	0.03546	1.451	0.6021	0.03688	1.540	0.5551
0.030	0.03377	1.204	0.7760	0.03625	1.344	0.6693	0.03815	1.453	0.6026	0.03970	1.543	0.5556
0.035	0.03957	1.206	0.7767	0.04257	1.347	0.6701	0.04487	1.457	0.6036	0.04675	1.549	0.5566
0.040	0.04537	1.208	0.7772	0.04890	1.350	0.6708	0.05160	1.461	0.6044	0.05381	1.554	0.5574
0.045	0.05117	1.209	0.7776	0.05522	1.352	0.6713	0.05833	1.464	0.6049	0.06087	1.557	0.5580
0.050	0.05697	1.210	0.7779	0.06155	1.354	0.6717	0.06505	1.466	0.6054	0.06792	1.560	0.5585
0.055	0.06277	1.211	0.7782	0.06787	1.355	0.6721	0.07178	1.468	0.6058	0.07498	1.562	0.5589
0.060	0.06857	1.212	0.7784	0.07420	1.356	0.6725	0.07851	1.469	0.6061	0.08204	1.564	0.5592

TABLE II

Weight and Diameter Ratios for Fully Developed Turbulence in Both Systems

(Same Absolute Roughness)

f_1	f_2	w_2/w_1	D_2/D_1	f_3	w_3/w_1	D_3/D_1	f_4	w_4/w_1	D_4/D_1	f_5	w_5/w_1	D_5/D_1
0.008	0.00835	1.168	0.7644	0.00855	1.279	0.6530	0.00871	1.365	0.5842	0.00884	1.436	0.5359
0.009	0.00941	1.169	0.7647	0.00966	1.282	0.6536	0.00985	1.368	0.5848	0.01000	1.439	0.5365
0.010	0.01048	1.170	0.7650	0.01078	1.284	0.6542	0.01100	1.371	0.5854	0.01117	1.442	0.5371
0.011	0.01155	1.171	0.7653	0.01190	1.286	0.6547	0.01216	1.373	0.5860	0.01236	1.446	0.5377
0.012	0.01263	1.172	0.7657	0.01303	1.287	0.6551	0.01332	1.376	0.5865	0.01355	1.448	0.5382
0.013	0.01371	1.173	0.7660	0.01416	1.289	0.3555	0.01449	1.378	0.5870	0.01475	1.451	0.5387
0.014	0.01480	1.175	0.7663	0.01529	1.290	0.6559	0.01567	1.380	0.5874	0.01596	1.454	0.5393
0.015	0.01588	1.175	0.7666	0.01644	1.292	0.6563	0.01685	1.382	0.5879	0.01718	1.457	0.5398
0.016	0.01697	1.176	0.7669	0.01759	1.294	0.6567	0.01804	1.384	0.5884	0.01840	1.459	0.5403
0.017	0.01807	1.177	0.7671	0.01874	1.295	0.6571	0.01924	1.386	0.5888	0.01964	1.462	0.5407
0.018	0.01917	1.178	0.7674	0.01990	1.297	0.6574	0.02044	1.388	0.5892	0.02089	1.464	0.5412

TABLE II (Continued)

f_1	f_2	w_2/w_1	D_2/D_1	f_3	w_3/w_1	D_3/D_1	f_4	w_4/w_1	D_4/D_1	f_5	w_5/w_1	D_5/D_1
0.019	0.02026	1.179	0.7677	0.02106	1.298	0.6578	0.02165	1.390	0.5896	0.02213	1.467	0.5416
0.020	0.02136	1.179	0.7679	0.02223	1.300	0.6582	0.02287	1.392	0.5900	0.02339	1.469	0.5420
0.022	0.02357	1.181	0.7684	0.02458	1.302	0.6589	0.02532	1.396	0.5907	0.02592	1.473	0.5428
0.024	0.02579	1.182	0.7689	0.02694	1.305	0.6595	0.02779	1.399	0.5914	0.02848	1.478	0.5436
0.026	0.02802	1.184	0.7693	0.02931	1.307	0.6601	0.03028	1.402	0.5921	0.03106	1.481	0.5444
0.028	0.03026	1.185	0.7697	0.03170	1.309	0.6607	0.03280	1.406	0.5928	0.03368	1.485	0.5451
0.030	0.03251	1.186	0.7701	0.03411	1.311	0.6612	0.03533	1.409	0.5934	0.03633	1.490	0.5458
0.035	0.03815	1.189	0.7710	0.04020	1.317	0.6625	0.04175	1.416	0.5950	0.04302	1.498	0.5474
0.040	0.04386	1.192	0.7720	0.04637	1.322	0.6637	0.04828	1.423	0.5964	0.04985	1.507	0.5490
0.045	0.04962	1.195	0.7728	0.05264	1.326	0.6649	0.05494	1.429	0.5977	0.05685	1.515	0.5505
0.050	0.05538	1.197	0.7735	0.05895	1.331	0.6660	0.06168	1.435	0.5990	0.06393	1.522	0.5518
0.055	0.06120	1.199	0.7742	0.06532	1.334	0.6670	0.06852	1.441	0.6002	0.07113	1.529	0.5530
0.060	0.06707	1.201	0.7749	0.07180	1.338	0.6680	0.07543	1.446	0.6013	0.07843	1.536	0.5542

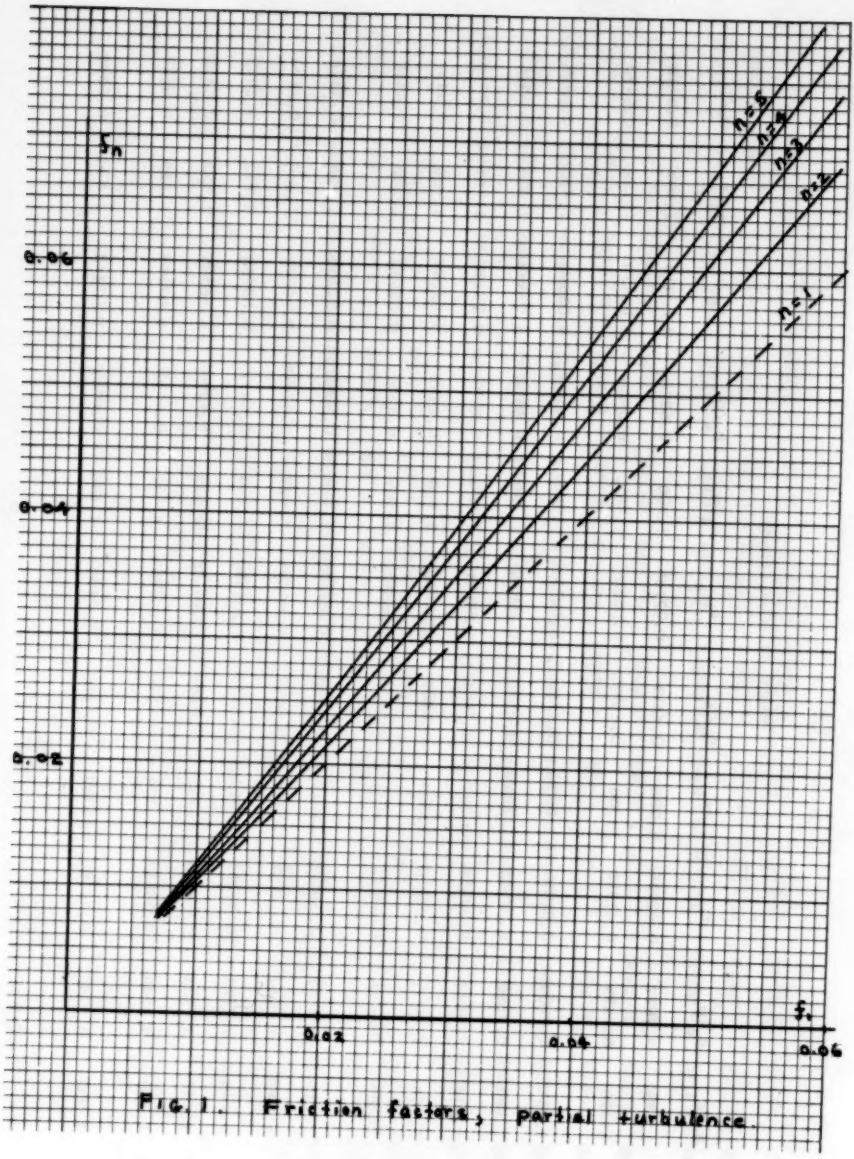

FIG. 1. Friction factors, partial turbulence.

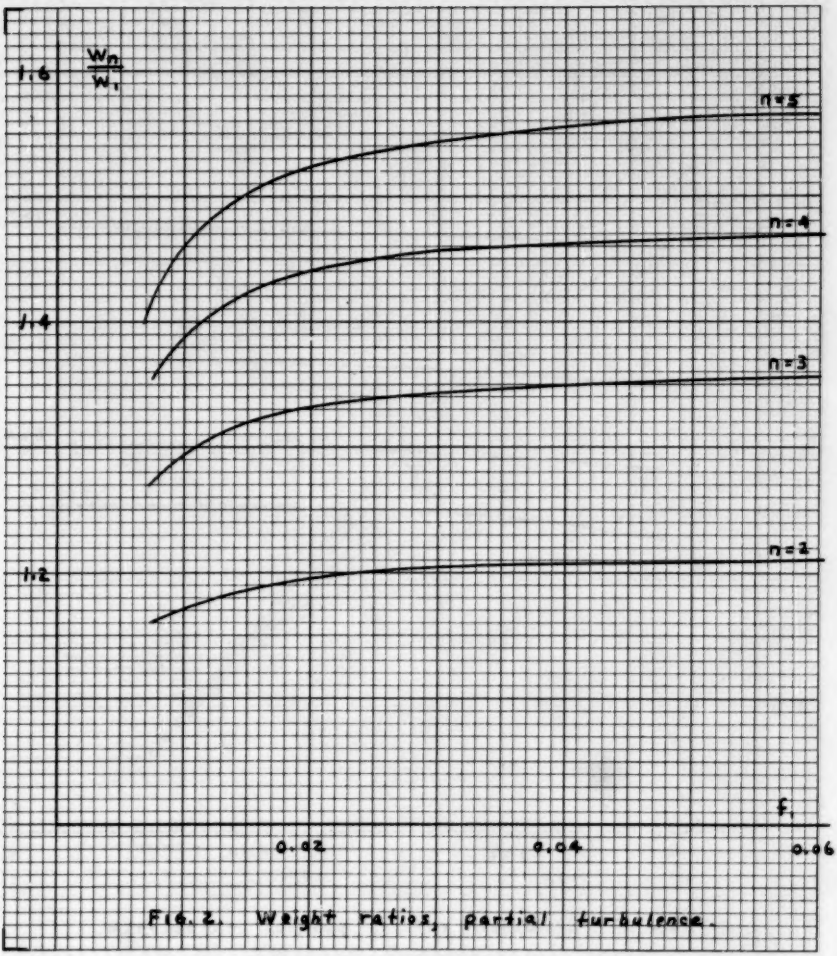

FIG. 2. Weight ratios, partial turbulence.

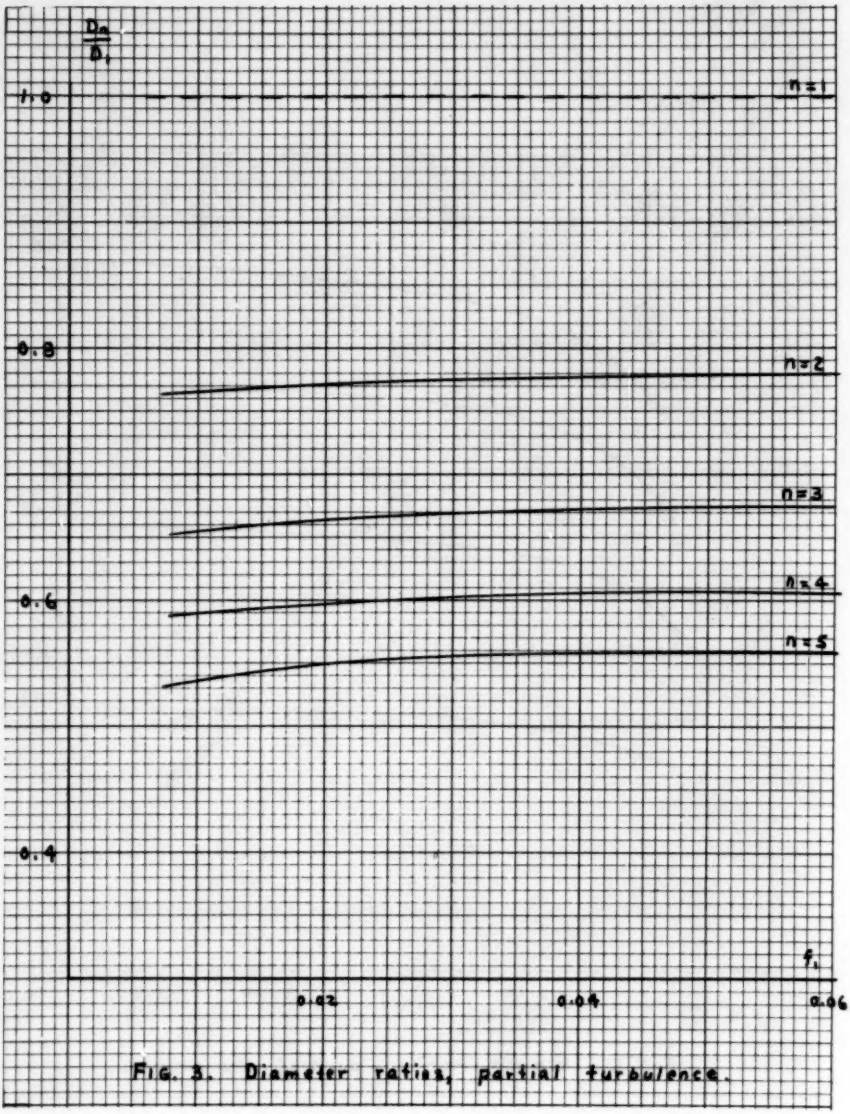

FIG. 3. Diameter ratios, partial turbulence.

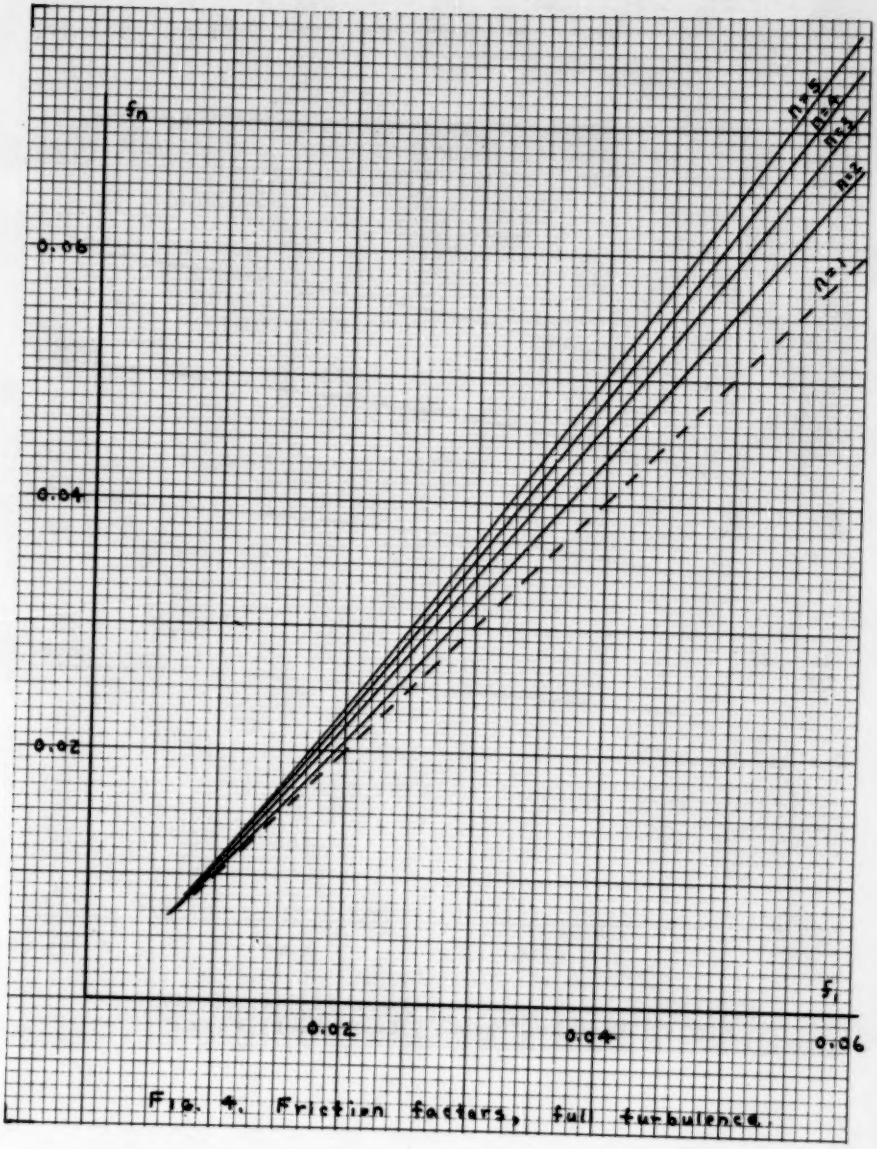

Fig. 4. Friction factors, full turbulence.

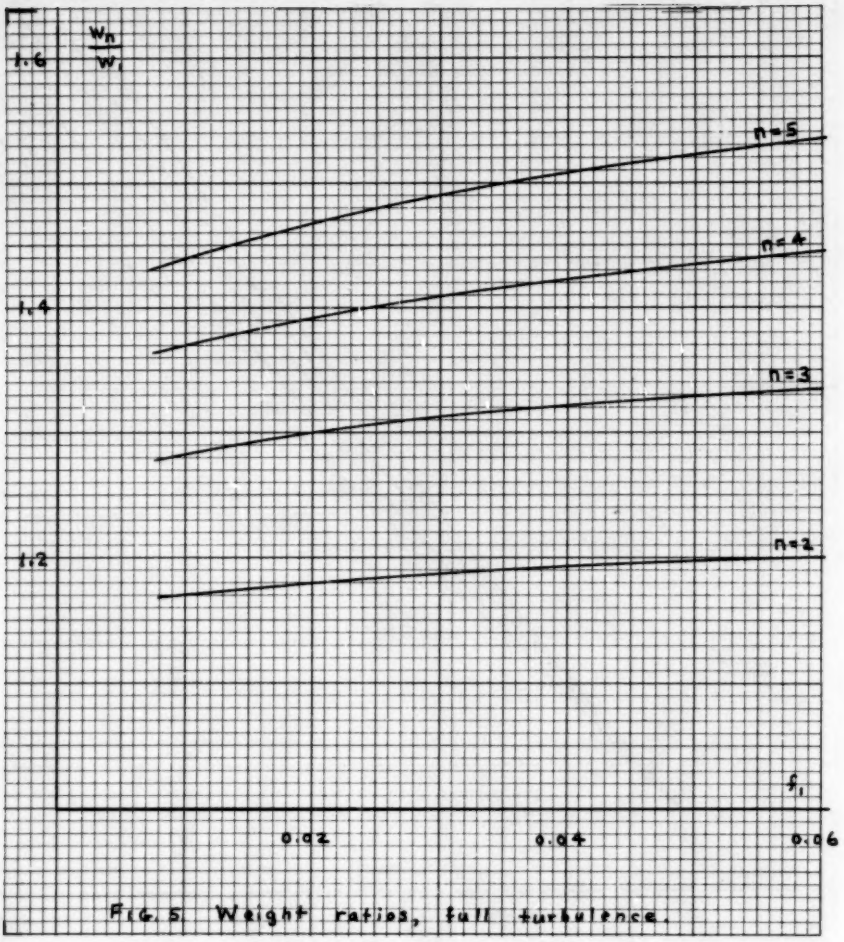

FIG. 5. Weight ratios, full turbulence.

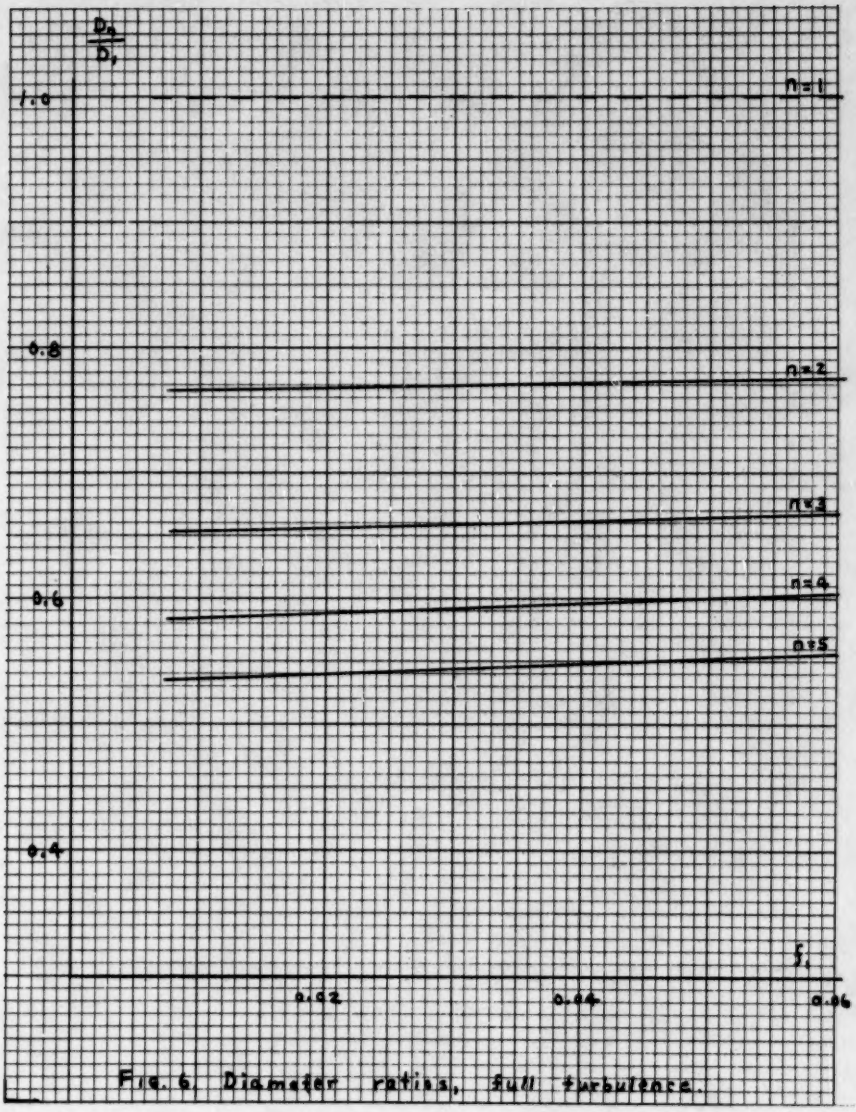

Fig. 6 Diameter ratios, full turbulence.

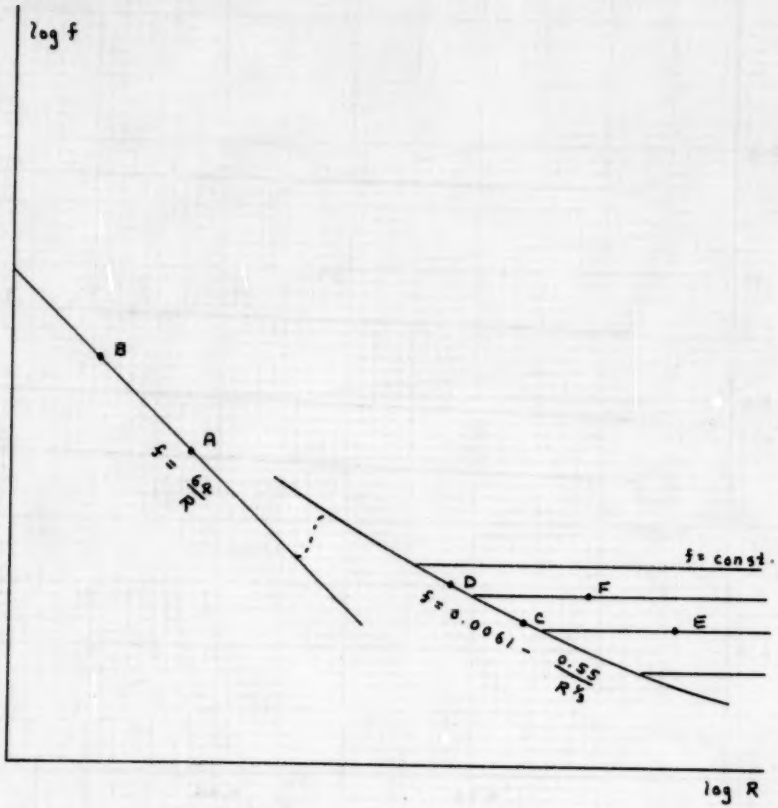

Fig. 7.